

A perfect-fluid spacetime for a slightly deformed mass

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We present approximate exterior and interior solutions of Einstein's equations which describe the gravitational field of a static deformed mass distribution. The deformation of the source is taken into account up to the first order in the quadrupole.

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To describe the gravitational field of a static axially symmetric mass distribution in general relativity, it is necessary to consider the multipole moments of the source. From a physical point of view, one expects that the quadrupole is the largest contributor and higher multipoles can be neglected in a first approximation. In this case, to describe the exterior field one can use, for instance, the exact quadrupole metric (q -metric)^{1,2}.

$$ds^2 = A^{1+q} dt^2 - A^{-q} \times \left[\left(1 + \frac{m^2 \sin^2 \theta}{r^2 A} \right)^{-q(2+q)} \left(\frac{dr^2}{A} + r^2 d\theta^2 \right) + r^2 \sin^2 \theta d\varphi^2 \right], \quad (1)$$

with $A = 1 - 2m/r$, which has been shown to be the simplest generalization of the Schwarzschild metric containing a quadrupole parameter q . Considering the quadrupole up to the first order only, we obtain

$$ds^2 = A(1 + q \ln A) dt^2 - r^2 \sin^2 \theta (1 - q \ln A) d\varphi^2 - \left[1 + q \ln A - 2q \ln \left(A + \frac{m^2}{r^2} \sin^2 \theta \right) \right] \left(\frac{dr^2}{A} + r^2 d\theta^2 \right). \quad (2)$$

This is an approximate solution of Einstein's vacuum equations up to the first order in q . The total mass of the spacetime turns out to be $M_0 = m(1 + q)$ and the quadrupole moment is $M_2 = -(2/3)qm^3$.

The interior solution can be generated by using the method proposed recently in Ref. 3. We obtain

$$ds^2 = e^{2\psi_0}(1 + 2\tilde{q}\psi_0)dt^2 - e^{-2\psi_0}(1 - 2\tilde{q}\psi_0) \times \left[e^{2\gamma_0}(1 + 4\tilde{q}\gamma_0 + \tilde{q}\gamma_1) \left(\frac{dr^2}{r^2 f^2(r)} + r^2(\sin^2 \theta - \tilde{q} \sin \theta \cos \theta) d\varphi^2 \right) \right], \quad (3)$$

$$e^{2\psi_0} = \frac{3}{2}f(R) - \frac{1}{2}f(r), \quad f(r) = \sqrt{1 - \frac{2\tilde{m}r^2}{R^3}}, \quad e^{\gamma_0} = r e^{2\psi_0}, \quad (4)$$

$$\gamma_1 = -2 \int \frac{1 + 4\pi \sin^2 \theta r^2 p_0}{r f^2(r)(1 + r\psi_{0,r}) \sin^2 \theta + \frac{r}{1+r\psi_{0,r}} \cos^2 \theta} dr + \kappa, \quad (5)$$

$$\psi_{0,r} = \frac{2\tilde{m}r}{R^3 f(r)[3f(R) + f(r)]}, \quad (6)$$

where \tilde{m} , \tilde{q} , R and κ are real constants. This is an interior solution up to the first order in \tilde{q} for a perfect fluid with density and pressure

$$\rho = \rho_0[1 + \tilde{q}(1 + \psi_0 - 4\gamma_0 - \gamma_1)], \quad \rho_0 = \text{const.} \quad (7)$$

$$p = p_0[1 + \tilde{q}(1 + \psi_0 - 4\gamma_0 - \gamma_1)], \quad p_0 = \rho_0 \frac{f(r) - f(R)}{3f(R) - f(r)}, \quad (8)$$

respectively. In the limiting case $\tilde{q} \rightarrow 0$, the metric (3) represents a perfect fluid with constant density ρ_0 and pressure p_0 as given in Eq.(8). If $\tilde{m} = m$, this particular solution can be matched with the exterior Schwarzschild metric along a sphere of radius R .

In the general case $\tilde{q} \neq 0$, a more detailed analysis must be carried out in order to match the above approximate interior solution with the approximate exterior q -metric given in Eq.(2). First, the matching surface must be established. Then, the matching conditions must be imposed for all metric components. This would imply a relationship between the exterior parameters m and q and the interior parameters \tilde{m} , \tilde{q} , ρ_0 and κ . This result will be presented elsewhere.

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